**Compensation Limits for Basic PID Loops**

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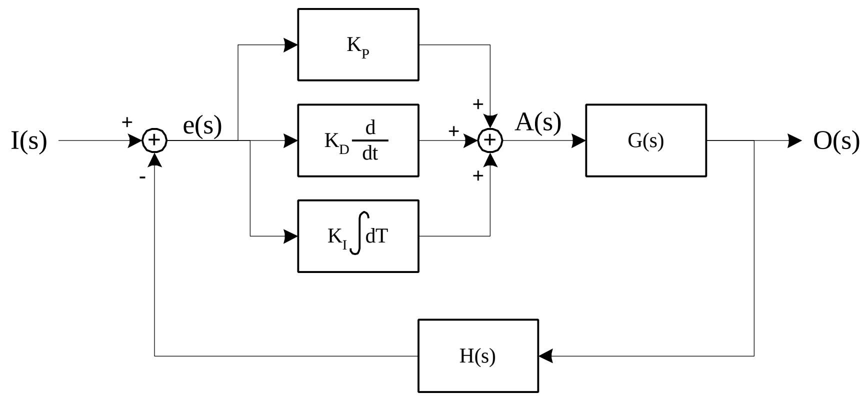
## Scope

This document is a simple analysis of PID control loops and the limits of compensation they offer. The goal of this analysis is to determine the limits of stability that can easily be achieved using a simple PID control system.

## Overall Configuration

For a typical PID loop of the overall configuration as shown in Figure 1, the system can be shown to obey the equations and transfer functions as shown in the following equations.




G(s) and H(s) are the LaPlace transforms of the forward path and feedback path (respectively) transfer functions outside the PID components.

**Figure 1**

Basic Open Loop Transfer Function:

$$OLTF(s) = (K_P + K_D s + \frac{K_I}{s})\, G(s)\, H(s)$$

$$OLTF(s) = (\frac{K_P s + K_D s^2 + K_I}{s})\, G(s)\, H(s)$$

Unifying into a common denominator and a standard form:

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) G(s)\, H(s)$$

**Case I:** $G(s) = \dfrac{K_G}{\dfrac{s}{w_G} + 1} \quad H(s) = K_H$

**1 pole in forward path, pure gain in feedback path**

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) \left( \frac{K_G}{\frac{s}{w_G} + 1} \right) K_H$$

To achieve an unconditionally stable Type I system, force the following conditions:

$$K_D = 0, \quad \frac{K_I}{K_P} = w_G$$

This will result in cancellation of the pole and a net OLTF of:

$$OLTF(s) = \frac{K_I K_G K_H}{s}$$

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**Case II:** $G(s) = \dfrac{K_G}{\dfrac{s}{w_G} + 1} \quad H(s) = \dfrac{K_H}{\dfrac{s}{w_H} + 1}$

**1 pole in forward path, 1 pole in feedback path**

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) \left( \frac{K_G}{\frac{s}{w_G} + 1} \right) \left( \frac{K_H}{\frac{s}{w_H} + 1} \right)$$

Combining the poles:

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) \left( \frac{K_G K_H}{\frac{s^2}{w_G w_H} + s\left( \frac{1}{w_G} + \frac{1}{w_H} \right) + 1} \right)$$

To achieve an unconditionally stable Type I system, force the following conditions:

$$\frac{K_D}{K_I} = \frac{1}{w_G w_H}, \quad \frac{K_P}{K_I} = \frac{1}{w_G} + \frac{1}{w_H}$$

This will result in cancellation of the pole and a net OLTF of:

$$OLTF(s) = \frac{K_I K_G K_H}{s}$$

**Case III:** $G(s) = \dfrac{K_G}{\dfrac{s}{w_G} + 1} \quad H(s) = \dfrac{K_H}{\dfrac{s}{w_H} + 1}$

## A pair of separable or inseparable poles in forward path, 1 pole in feedback path

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) \left( \frac{K_G}{\frac{s^2}{w_G{}^2} + \frac{2z}{w_G} s + 1} \right) \left( \frac{K_H}{\frac{s}{w_H} + 1} \right)$$

To optimize the system, force the following conditions:

$$\frac{K_D}{K_I} = \frac{1}{w_G{}^2}$$

$$\frac{K_P}{K_I} = \frac{2z}{w_G}$$

This will result in cancellation of the pole pair and a net OLTF of:

$$OLTF(s) = \frac{K_I K_G K_H}{s\left( \frac{s}{w_H} + 1 \right)}$$

Optimizing this system for maximum loop bandwidth while maintaining stability would be to increase $K_G$ and/or $K_H$ in order to force loop cross over to occur with 135º phase shift.

At s = j$w_H$

$$1 = \frac{K_I K_G K_H}{w_H \frac{1}{\sqrt{2}}}$$

$$K_I K_G K_H = \frac{w_H}{\sqrt{2}}$$

If this criteria is met, then loop bandwidth is maximized while preserving stability.

**Case IV:** $G(s) = \frac{K_G}{s} \quad H(s) = K_H$

**A single integrator in forward path**

$$OLTF(s) = K_I \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s} \right) \left( \frac{K_G}{s} \right) K_H$$

$$OLTF(s) = K_I K_G K_H \left( \frac{\frac{K_D}{K_I} s^2 + \frac{K_P}{K_I} s + 1}{s^2} \right)$$

This system will be conditionally stable. One approach to forcing stability is to re-arrange the equation so that it can be seen what happens if the PID integrator function is set to zero.

$$OLTF(s) = K_G K_H \left( \frac{K_D s^2 + K_P s + K_I}{s^2} \right)$$

if $K_I = 0$, then:

$$OLTF(s) = K_G K_H \left( \frac{K_D s + K_P}{s} \right)$$

$$OLTF(s) = K_P K_G K_H \left( \frac{\frac{K_D}{K_P} s + 1}{s} \right)$$

## Conclusions

At best, a PID loop can only provide unconditional closed loop stability if the system has 2 poles or less. If a sampled data system is present, as with most system using an A/D and D/A process, then the effects of sampling must be accounted for (sampling will appear as a fixed time delay or phase shift). This will severely limit the maximum bandwidth achievable as opposed to a standard control system using pole/zero compensation. PID loops are recommended only when 2 or less poles are involved.